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Differentiating and reducing,

$$(r^3\sqrt{2}-2ar^2-2a^3)\sqrt{(2a^2+r^2)}+r^4\sqrt{2}+2a^4\sqrt{2}-4a^3r-2ar^3=0.$$

Let $r^3/a^3=y^3$.

$$\therefore (y^3\sqrt{2}-2ay^2-2)\sqrt{(2+y^2)}+y^4 2\sqrt{2}-4y-2y^3=0.$$

$\therefore y=0$ or $a=\infty$ for the minimum effort.

b) $CB_2=\dfrac{r+\sqrt{(2a^2+r^2)}}{\sqrt{2}}$, $A_2B_2=\sqrt{(a^2+r^2)}$, $A_1A_2=r\sqrt{(2-\sqrt{2})}$.

$$A_1B_2=\sqrt{[a^2+2r^2+r\sqrt{(2a^2+r^2)}]}=A_2B_2+A_1A_2=\sqrt{(a^2+r^2)}+r\sqrt{(2-\sqrt{2})}.$$

Let $a^2+r^2=y^2$.

$$\therefore \sqrt{[y^2+r^2+r\sqrt{(2y^2-r^2)}]}=y+r\sqrt{(2-\sqrt{2})}$$

$$\text{or } r(1-\sqrt{2})+2y\sqrt{(2-\sqrt{2})}=\sqrt{(2y^2-r^2)}.$$

$$\therefore y=\frac{r\sqrt{(2-\sqrt{2})}}{\sqrt{(2)}-1},\quad a=\frac{r}{\sqrt{[\sqrt{(2)}-1]}}.$$

c) $x/a-y/r=1$ is the equation to A_1B_1 ....(1).

$rx+y\sqrt{(2a^2+r^2)}-r^2\sqrt{2}-r\sqrt{2}\sqrt{(2a^2+r^2)}=0$ is equation to A_2B_2 ....(2).

From (1), $a=\dfrac{rx}{r+y}$; this in (2) gives

$$(y-r\sqrt{2})^2[2x^2+(r+y)^2]=(r+y)^2(r\sqrt{2}-x)^2 \text{ for first locus.}$$

$$\frac{2ax}{r\sqrt{2}[r+\sqrt{(2a^2+r^2)}]}+\frac{y\sqrt{2}}{r+\sqrt{(2a^2+r^2)}}=1 \text{ is equation to } A_4B_2 \text{ ....(3).}$$

$$\therefore 2ax+2ry=r\sqrt{2}[r+\sqrt{(2a^2+r^2)}] \text{ ....(4).}$$

$$[r^2-2a^2+r\sqrt{(2a^2+r^2)}]y=r[r+\sqrt{(2a^2+r^2)}]x-ar[r+\sqrt{(2a^2+r^2)}]$$

is equation to A_3B_1 ....(5).

From (4),

$$a=\frac{r^2x\sqrt{2}-2rxy\pm r\sqrt{[4x^2y^2+2r^2x^2-2\sqrt{(2)}r^3y]}}{2x^2-r^2}.$$

Substituted in (5) this gives the second locus.

171. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Prove that the electrical capacity of an oblate ellipsoid of revolution is $\sqrt{(a^2-b^2)}/\cos^{-1}(b/a)$, where a and b are the equatorial and polar semi-diameters.

I. Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

Suppose the thickness of the shell to be $m\phi$, then the force due to the shell at any point P, external, is $4\pi\rho m\phi$. The force due to the shell is obtained from $-\dfrac{dn}{du}=\dfrac{4\pi\rho m\phi abc}{\sqrt{[(a^2+l)(b^2+l)(c^2+l)]}}$, where dn is an element of the normal at ρ.

Taking dx, dy, and dz in the direction of the normal we get from the differentiation of $\frac{x^2}{a^2+l}+\frac{y^2}{b^2+l}+\frac{z^2}{c^2+l}=1$, $dl=2\phi dn$.

Hence $dv=\frac{2\pi\rho mabcdl}{\sqrt{[(a^2+l)(b^2+l)(c^2+l)]}}$, from which we get

$$v=2\pi\rho mabc\int_l^\infty \frac{dl}{\sqrt{[(a^2+l)(b^2+l)(c^2+l)]}}.$$

Replacing $4\pi\rho mabc$, the mass of the shell, by Q in the electrical case

$$v=\frac{Q}{2}\int_l^\phi \frac{dl}{\sqrt{[(a^2+l)(b^2+l)(c^2+l)]}}$$

$$\text{and } \frac{Q}{v}=\left[\tfrac{1}{2}\int_0^\infty \frac{dl}{\sqrt{[(a^2+l)(b^2+l)(c^2+l)]}}\right]^{-1}.$$

Since $l=0$ and $a=c>b$, we get, upon integrating,

$$\frac{Q}{\quad}=\sqrt{(a^2-b^2)}\Big/\left(\tfrac{1}{2}\pi-\tan^{-1}\frac{b}{\sqrt{(a^2-b^2)}}\right)=\sqrt{(a^2-b^2)}\Big/\left(\tfrac{1}{2}\pi-\sin^{-1}\frac{b}{a}\right)$$

$$=\sqrt{(a^2-b^2)}/\cos^{-1}(b/a).$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The electrification at any point is proportional to the length of the perpendicular from the center on the tangent plane at that point. Let us denote it by mp. We must have $\int mpdS$ equal to the charge Q.

But $\int pdS=4\pi a^2b$. $\therefore m=Q/4\pi a^2b$.

The potential V at any point in the interior of the conductor is constant. At the center we have

$$V=\int\frac{1}{r}\frac{Qp}{4\pi a^2b}2\pi r\sin\theta ds=\frac{Q}{2a^2b}\int p\sin\theta ds=\frac{Q}{2a^2b}\int_0^\pi r^2\sin\theta d\theta$$

$$=\tfrac{1}{2}Qb\int_0^\pi\frac{\sin\theta d\theta}{a^2\cos^2\theta+b^2\sin^2\theta}=\frac{Q}{2\sqrt{(a^2-b^2)}}\tan^{-1}\left[\frac{\sqrt{(a^2-b^2)}}{b}\cos\theta\right]_\pi^0$$

$$=\frac{Q}{\sqrt{(a^2-b^2)}}\tan^{-1}\left[\frac{\sqrt{(a^2-b^2)}}{b}\right]=\frac{Q}{\sqrt{(a^2-b^2)}}\cos^{-1}(b/a).$$

$\therefore$ The capacity $Q/V=\sqrt{(a^2-b^2)}/\cos^{-1}(b/a)$.